Hitting minors on bounded treewidth graphs

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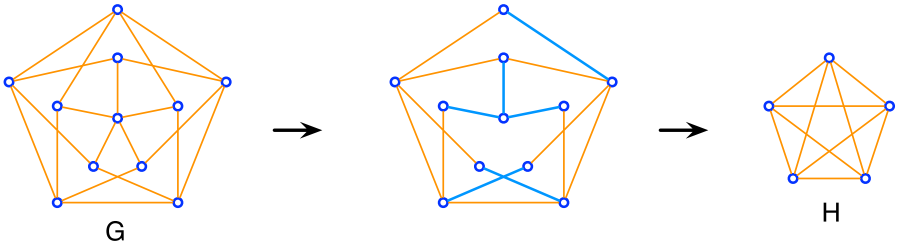
[arXiv 1704.07284]

Minors and topological minors



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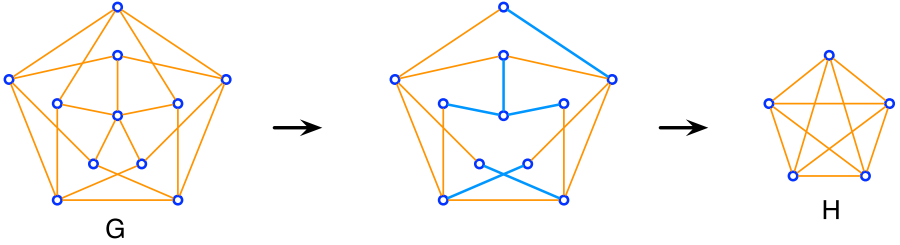

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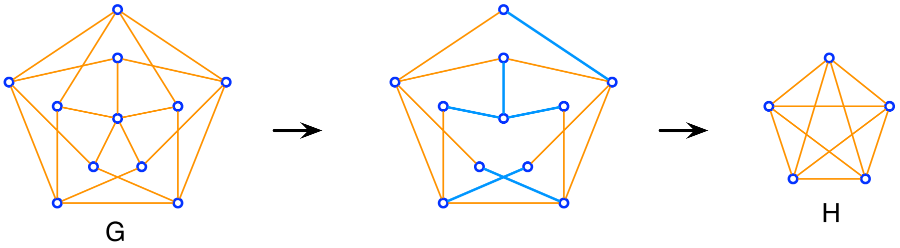

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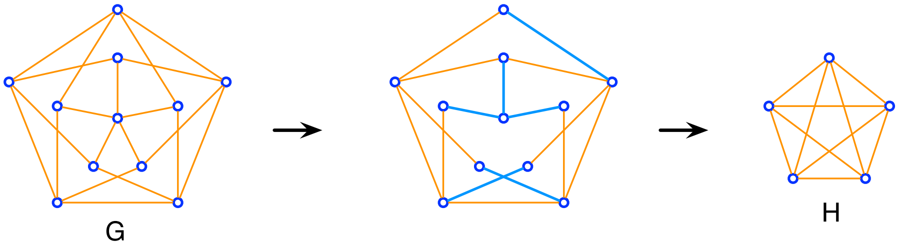

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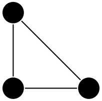
- Therefore: $\boxed{H \text{ topological minor of } G \not\Leftarrow H \text{ minor of } G}$

Example of a 2-tree:

A k-tree is a graph that can be built starting from a $(k+1)$-clique and then iteratively adding a vertex connected to a k-clique.



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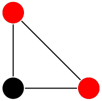

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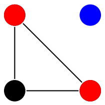

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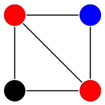

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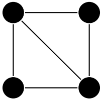

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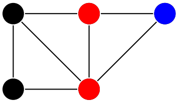

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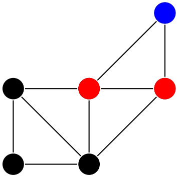

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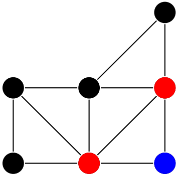

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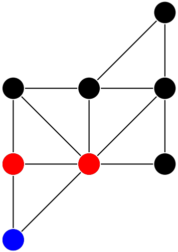

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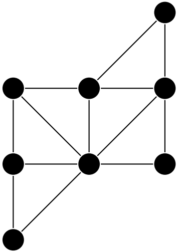

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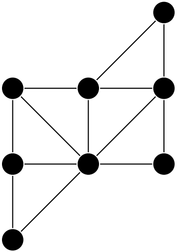

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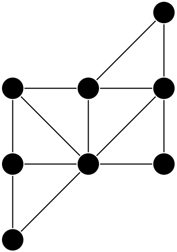

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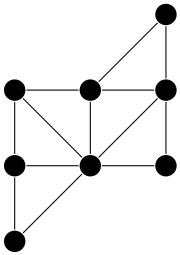

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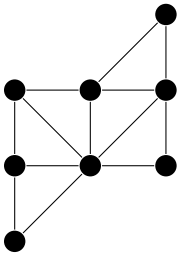

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Construction suggests the notion of tree decomposition: small separators.

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3. Treewidth behaves very well algorithmically...

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Monadic Second Order Logic (MSOL):
Graph logic that allows quantification over sets of vertices and edges.

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Theorem (Courcelle, 1990)

Every problem expressible in MSOL can be solved in time $f(\mathrm{tw}) \cdot n$ on graphs on n vertices and treewidth at most tw.

In parameterized complexity: FPT parameterized by treewidth.

Examples: Vertex Cover, Dominating Set, Hamiltonian Cycle, Clique, Independent Set, k-Coloring for fixed k, ...

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Remark: Algorithms parameterized by treewidth appear very often as a "black box" in all kinds of parameterized algorithms.

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But for the so-called connectivity problems, like LONGEST PATH or STEINER TREE, the "natural" DP algorithms provide only time

$$2^{\mathcal{O}(\mathrm{tw} \cdot \log \mathrm{tw})} \cdot n^{\mathcal{O}(1)}.$$

(Single-exponential algorithms on sparse graphs)

On topologically structured graphs (planar, surfaces, minor-free), it is possible to solve connectivity problems in time $2^{\mathcal{O}(\text{tw})} \cdot n^{\mathcal{O}(1)}$:

- Planar graphs: [Dorn, Penninkx, Bodlaender, Fomin. 2005]
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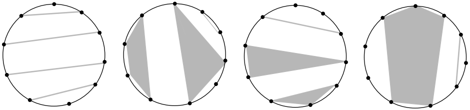
Main idea special type of decomposition with nice topological properties:

partial solutions $\Longleftrightarrow$ non-crossing partitions



$$\mathrm{CN}(k) = \frac{1}{k+1}\binom{2k}{k} \sim \frac{4^k}{\sqrt{\pi}k^{3/2}} \leq 4^k.$$

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Deterministic algorithms with algebraic tricks: [Bodlaender, Cygan, Kratsch, Nederlof. 2013]

Representative sets in matroids: [Fomin, Lokshtanov, Saurabh. 2014]

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There are other examples of such problems...

The $\mathcal{F}$-M-DELETION problem

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- $\mathcal{F} = \{K_5, K_{3,3}\}$: VERTEX PLANARIZATION.

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- $\mathcal{F} = \{K_5, K_{3,3}\}$: Vertex Planarization.
  Solvable in time $2^{\Theta(\mathrm{tw} \cdot \log \mathrm{tw})} \cdot n^{\mathcal{O}(1)}$. [Jansen, Lokshtanov, Saurabh. 2014 + Pilipczuk. 2017]

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FPT by Courcelle's Theorem.

Goal of this project

Objective

Determine, for every fixed $\mathcal{F}$, the (asymptotically) smallest function $f_{\mathcal{F}}$ such that $\mathcal{F}$-M-Deletion/$\mathcal{F}$-TM-Deletion can be solved in time

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- We do not want to optimize the degree of the polynomial factor.
- We do not want to optimize the constants.
- Our hardness results hold under the ETH.

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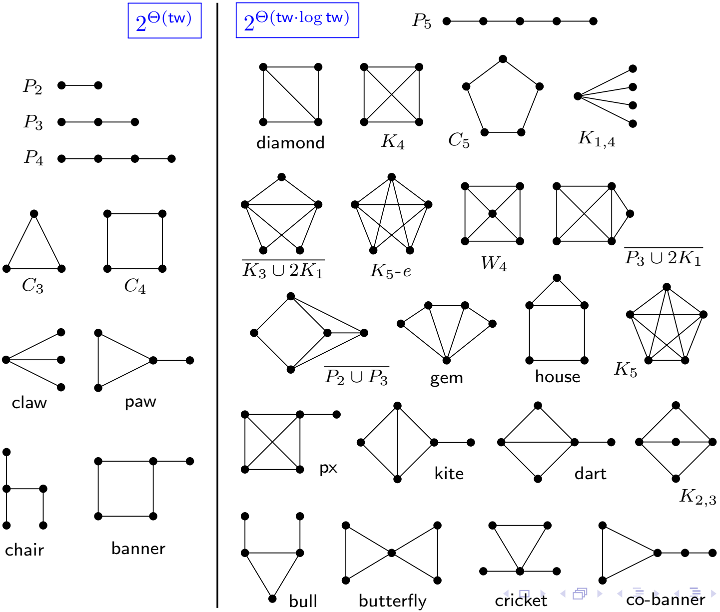
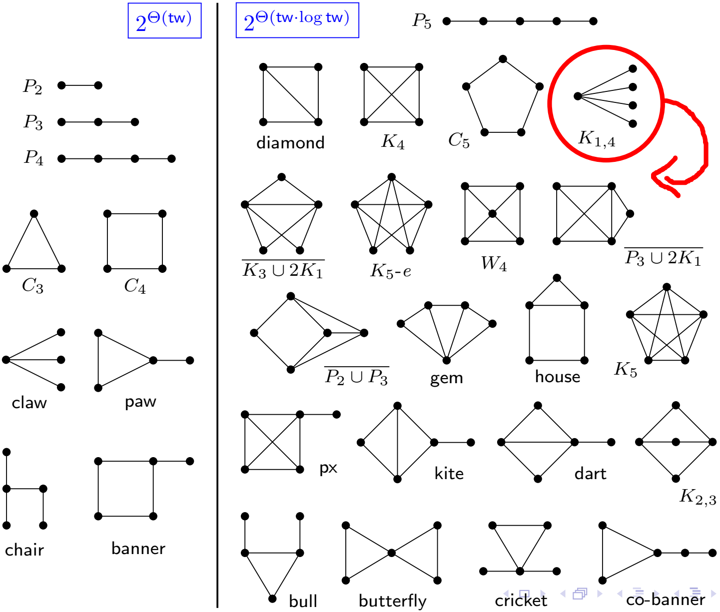
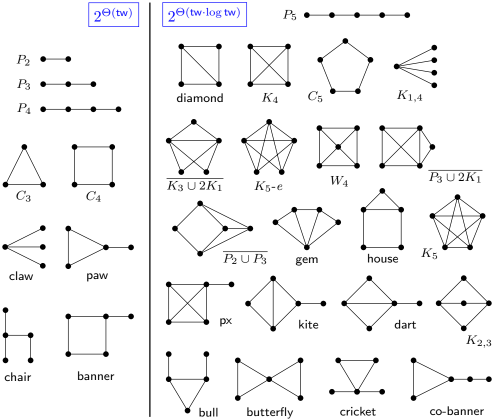
Complexity of hitting a single minor H



For topological minors, there (at least) one change



A compact statement for small planar minors



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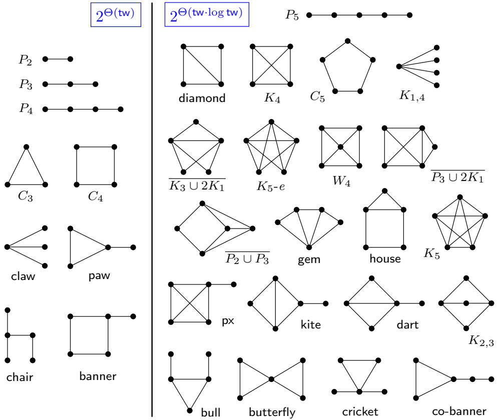

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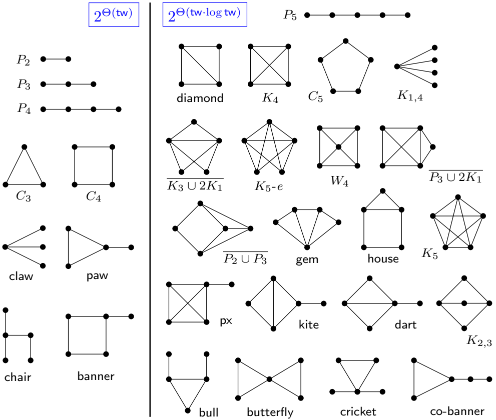

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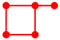
- All the graphs on the left are minors of  (called the banner)
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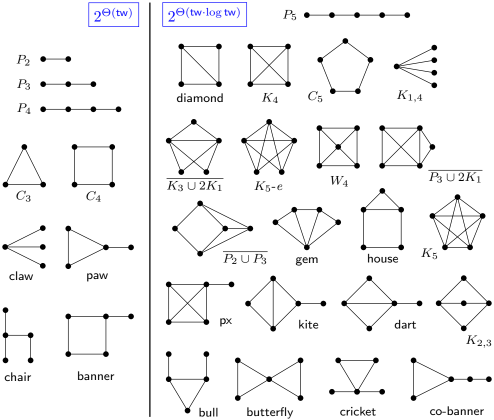

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- All the graphs on the right are not minors of  ... except P_5.

A dichotomy for hitting connected minors

We can prove that any connected H with $|V(H)| \geq 6$ is $\boxed{\text{hard}}$:
$\{H\}$-M-DELETION cannot be solved in time $2^{o(\mathrm{tw} \cdot \log \mathrm{tw})} \cdot n^{\mathcal{O}(1)}$ under the ETH.

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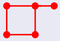
- $2^{\mathcal{O}(\mathrm{tw})} \cdot n^{\mathcal{O}(1)}$, *if* $H \preceq_{\mathrm{m}}$  *and* $H \neq P_5$.

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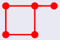
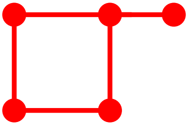
- $2^{\mathcal{O}(\mathrm{tw})} \cdot n^{\mathcal{O}(1)}$, *if* $H \preceq_{\mathrm{m}}$  *and* $H \neq P_5$.
- $2^{\mathcal{O}(\mathrm{tw} \cdot \log \mathrm{tw})} \cdot n^{\mathcal{O}(1)}$, *otherwise.*

In both cases, the running time is asymptotically optimal under the ETH.

Why the banner??



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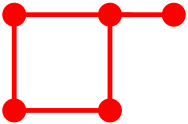

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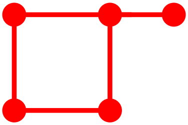

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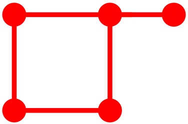

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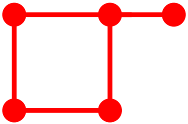

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- If the characterization of the allowed connected components is enriched in some way, such as restricting the length of the allowed cycles or forbidding certain degrees, the problem becomes harder.

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3. Lower bounds under the ETH
   - $2^{o(\mathrm{tw})}$ is "easy".
   - $2^{o(\mathrm{tw}\cdot\log \mathrm{tw})}$ is much more involved and we get ideas from:

     [Lokshtanov, Marx, Saurabh. 2011] [Marcin Pilipczuk. 2017] [Bonnet, Brettell, Kwon, Marx. 2017]

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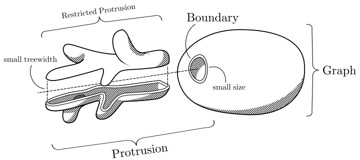

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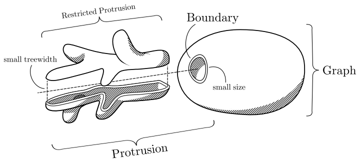

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  Extra: Bidimensionality, irrelevant vertices, protrusion decomposition...

skip

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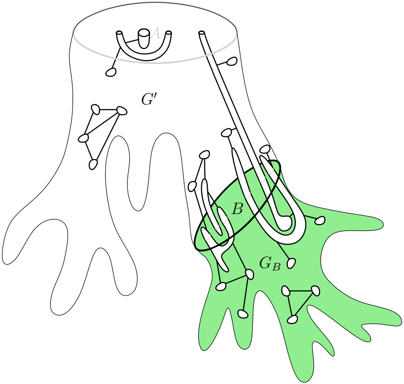

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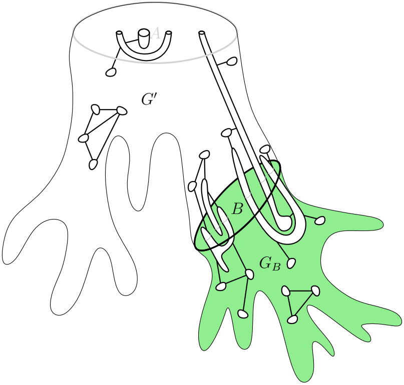

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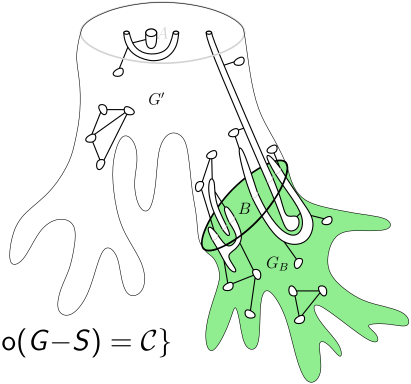

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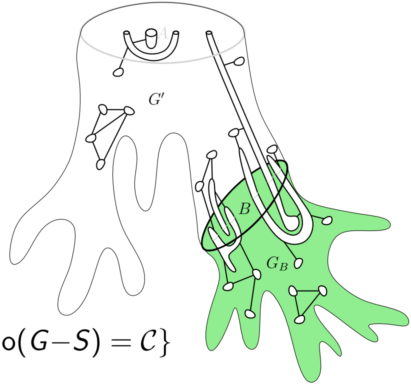

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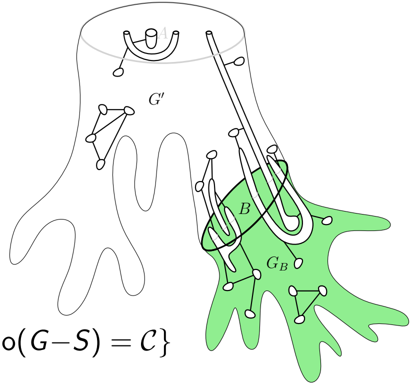

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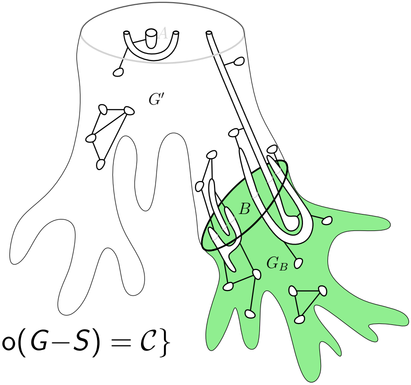

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skip

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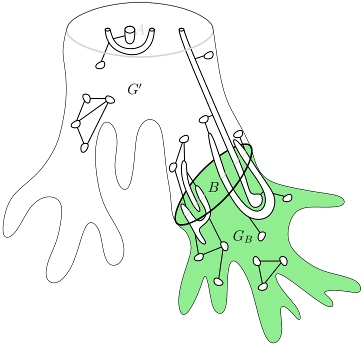

Algorithm for a connected and planar collection $\mathcal{F}$

- For a fixed $\mathcal{F}$, we define an equivalence relation $\equiv^{(\mathcal{F},t)}$ on t-boundaried graphs:

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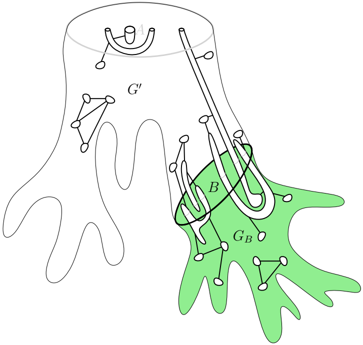

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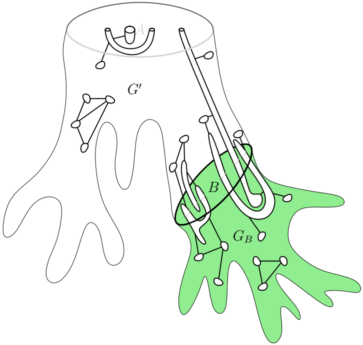

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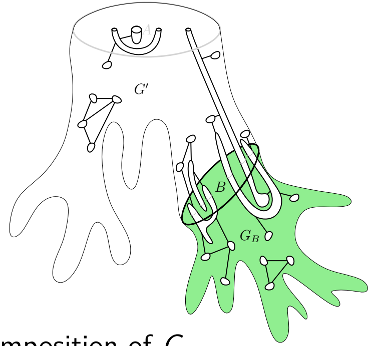

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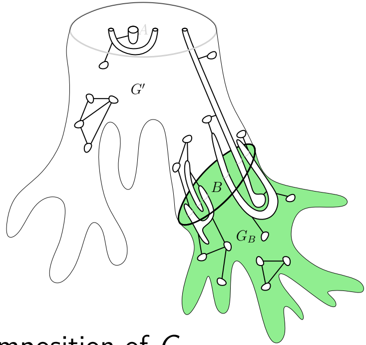

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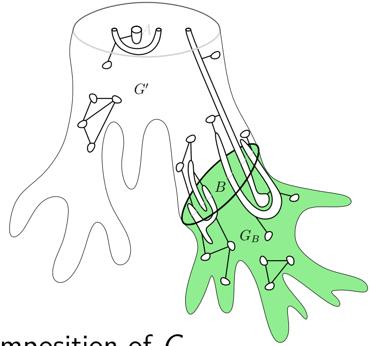

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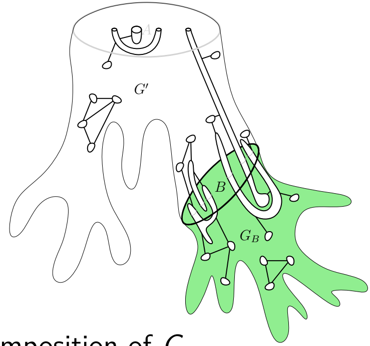

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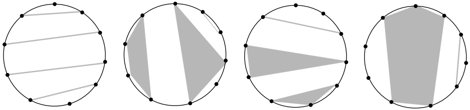

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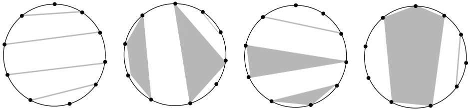

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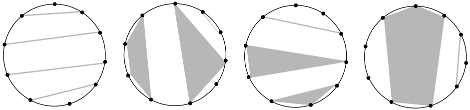

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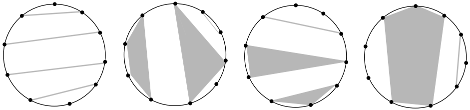

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Gràcies!