Characterization, probe and sandwich problems on a generalization of threshold graphs

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Joint work with Fernanda Couto, Luerbio Faria, Sylvain Gravier and Sulamita Klein

February, 2019

Graph Classes

Some graph classes you probably already heard of:

- Trees
- Interval (Jayme's talk)
- Bipartite
- Complete graphs
- Cycles
- . . . (more than 1500 graph classes registered in http://graphclasses.org)

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- May have strong structural properties, with algorithmic applications
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- Sometimes a particular case is a key step for understanding a problem
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Cographs

Definition 1

A cograph can be defined recursively as follows:

1. The trivial graph K_1 is a cograph;
2. If $G_1, G_2, \ldots, G_p$ are cographs, then $G_1 \cup G_2 \cup \ldots \cup G_p$ is a cograph,
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Cographs

Theorem 2

A cograph is a graph without induced P_4's, i.e, induced paths with 4 vertices.

D. G. Corneil and H. Lerchs and L. Stewart Burlingham
Complement reducible graphs
Discrete Applied Mathematics, 3, 1981, pp. 163-174.

(k, ℓ)-Graphs

Definition 3

A graph is (k, ℓ) if its vertex set can be partitioned into at most k independent sets and ℓ cliques.

Some well-known special cases: (0,1), (2,0), (1,1).

A. Brandstädt
Partitions of graphs into one or two independent sets and cliques.
Discrete Mathematics,152(1-3), 1996, pp. 47–54.

Threshold graphs

Definition 4

a graph is a threshold graph if there are a real number S and for each vertex v a weight $w(v)$ such that $uv \in E(G)$ if and only if $w(u) + w(v) > S$.

Theorem 5

A graph is a threshold graph if and only if it is both a cograph and a split graph.

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Threshold Graphs

They can be constructed from a K_1 by repeated applications of the following two operations:

1. Addition of a single isolated vertex to the graph.
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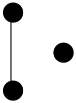

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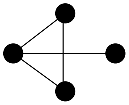

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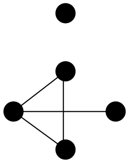

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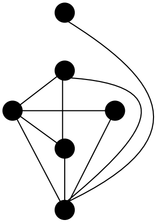

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R. Bravo and S. Klein and L. Nogueira
Characterizing (k, ℓ)-partitionable cographs
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Structural Characterizations of Cographs-$(2,1)$

Definition 6

Let x, y be vertices. We say that they have *nested neighborhoods* if $N(x) \subseteq N(y)$ or $N(y) \subseteq N(x)$.

Proposition 7

A cograph-$(2,1)$ is a cograph that can be partitioned into a bipartite graph B and a clique K.

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A structural characterization

Theorem 9

Let G be a graph. Then the following are equivalent.

1. *G is a cograph-$(2,1)$.*
2. *G can be partitioned into a collection of maximal bicliques $B = \{B_1, \ldots, B_l\}$ and a clique K such that $B_i = (X_i, Y_i)$ and $V(K)$ is the union of non-intersecting sets K^1 and K^2 such that the following properties hold:*

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- There are no edges between vertices of B_i and B_j for $i \neq j$;
- Let $L(v)$ be the list of bicliques in the neighborhood of v, $\forall v \in V$.
  $K^1 = \{v \in K | N(v) \cap B \subseteq B_1\}$ and
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  $K^{1,1} = \{v \in K^1 | vx \in E(G), \forall x \in X_1\}$ and
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- $G[X_1 \cup Y_1 \cup K^{1,1} \cup K^{1,2}]$ is the join of threshold graphs $(K^{1,1}, Y_1)$ and $(K^{1,2}, X_1)$;
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Let G be a graph. G is a cograph-$(2,1)$ if and only if G is either a join of two threshold graphs or it can be obtained from the join of two threshold graphs by the applications of any sequence of the following operations:

- *Disjoint union with a biclique;*
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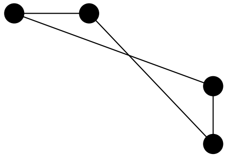

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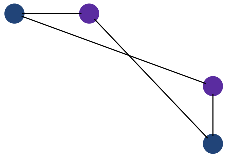

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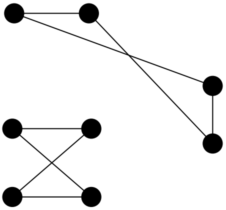

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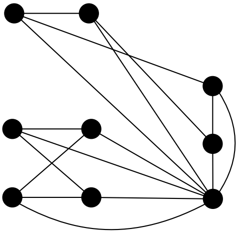

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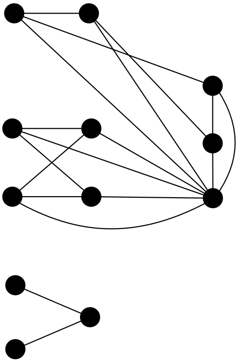

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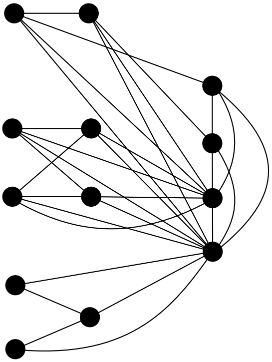

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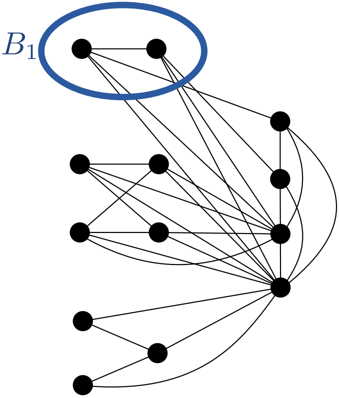

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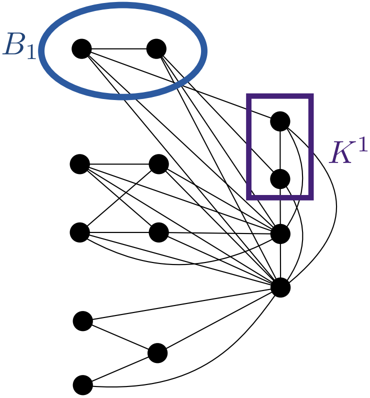

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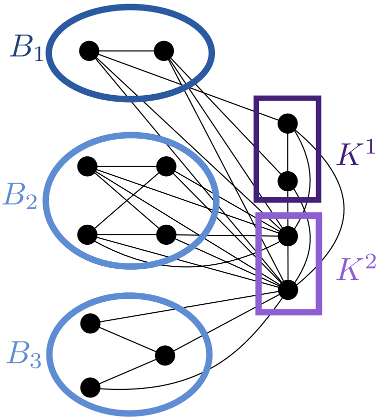

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Let $\mathcal{G}$ be a class of graphs. A graph $G = (V, E)$ is a *probe graph* if its vertex set can be partitioned into a set of *probes* P and an independent set of *nonprobes* N, such that G can be embedded in a graph of $\mathcal{G}$ by adding edges between certain nonprobes.

Probe Graphs

- If (P, N) is given and G is a probe graph, then $G = (P + N, E)$ is called *partitioned probe graph of* $\mathcal{G}$.
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From the literature we know that...

- The PROBE COGRAPH is solvable in polynomial time.

H. N. de Ridder
On probe classes of graphs
Ph.D. thesis, 2007.

From the literature we know that...

- The PP-(k, ℓ) is NP-complete for $k^2 + \ell^2 \geq 8$ and polynomial otherwise.

S. Dantas, L. Faria, C. M. H de Figueiredo, R. B. Teixeira
The generalized split problem
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- The PROBE (k, ℓ) is NP-complete for $k + \ell \geq 3$ and polynomial otherwise.

S. Dantas, L. Faria, C. M. H de Figueiredo, R. B. Teixeira
The (k, ℓ) unpartitioned probe problem NP-complete versus Polynomial dichotomy.
Submitted to IPL, 2014.

PP-COGRAPH-$(2,1)$

- Given a graph $G = (P + N, E)$. Is there a cograph-$(2,1)$ $G' = (P + N, E')$ such that $E' = E \cup \{xy | x, y \in N\}$ for certain $x, y \in N$?

Theorem 12

PP-COGRAPH-$(2,1)$ *can be solved in polynomial time.*

Main ingredient:

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PP-JOIN OF TWO THRESHOLD GRAPHS *can be solved in polynomial time.*

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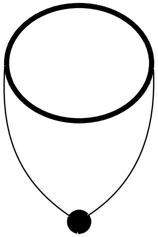

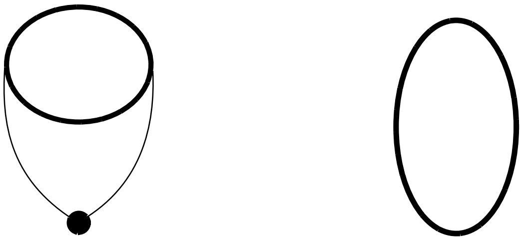

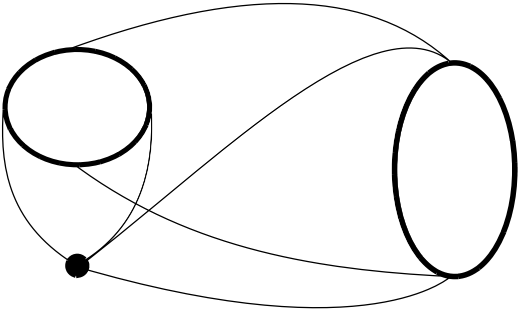

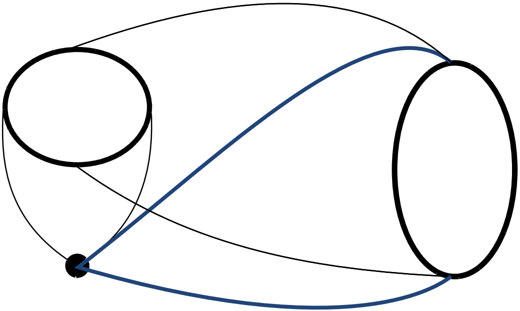

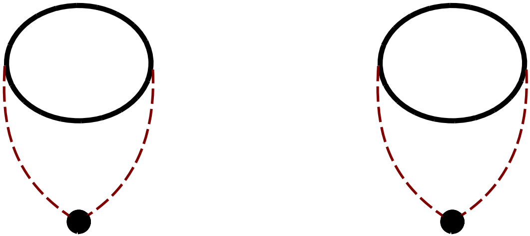

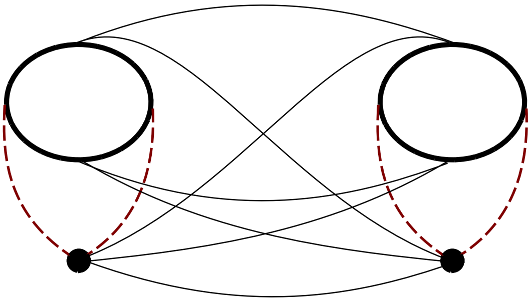

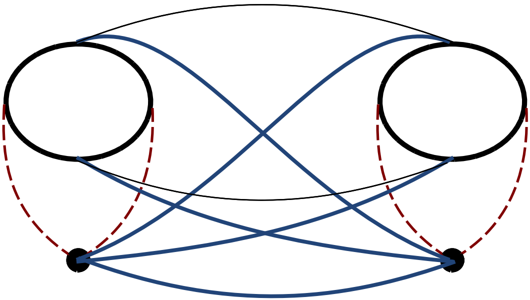

PP-COGRAPH-$(2, 1)$

Putting everything together:

- Easy to identify *probe* universal vertices
- Easy to identify *probe* isolated bicliques
- Easy to find an induced partition
- Solve 2 instances of PP-THRESHOLD

Definition

GRAPH SANDWICH PROBLEM FOR PROPERTY Π - (Π - SP)

Input: $G^1 = (V, E^1)$ and $G^2 = (V, E^2)$ such that $E^1 \subseteq E^2$.

Question: Is there a graph $G = (V, E)$ such that $E^1 \subseteq E \subseteq E^2$ and that G satisfies Π?

If such a graph exists, it is called *sandwich graph*.

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M.C. Golumbic, H. Kaplan and R. Shamir
Graph sandwich problems.
Journal of Algorithms, 19, 1995, pp. 449–473.

Definition

GRAPH SANDWICH PROBLEM FOR PROPERTY Π - (Π - SP)

Input: $G^1 = (V, E^1)$ and $G^2 = (V, E^2)$ such that $E^1 \subseteq E^2$.

Question: Is there a graph $G = (V, E)$ such that $E^1 \subseteq E \subseteq E^2$ and that G satisfies Π?

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JOIN OF TWO THRESHOLDS-SP

JOIN OF TWO THRESHOLDS GRAPH SANDWICH PROBLEM-(JTT-SP)

Input: $G^1 = (V, E^1)$ and $G^2 = (V, E^2)$ such that $E^1 \subseteq E^2$.

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Join of two thresholds-SP

Theorem 14

JTT-SP *is NP-complete.*

Proof's Idea. Polynomial-time reduction from the NP-complete problem MONOTONE NAE-3SAT.

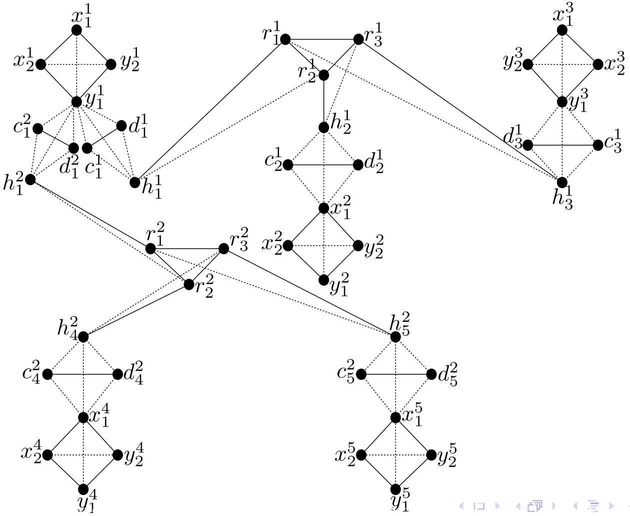
JOIN OF TWO THRESHOLDS-SP

Theorem 14

JTT-SP *is NP-complete.*

Proof's Idea. Polynomial-time reduction from the NP-complete problem MONOTONE NAE-3SAT.

$$I = (X, C) = (\{x_1, x_2, x_3, x_4, x_5\}, (x_1 \vee x_2 \vee x_3) \wedge (x_1 \vee x_4 \vee x_5))$$



COGRAPH-$(2,1)$ - SP

COGRAPH-$(2,1)$ GRAPH SANDWICH PROBLEM-(COGRAPH-$(2,1)$-SP)

Input: $G^1 = (V, E^1)$ and $G^2 = (V, E^2)$ such that $E^1 \subseteq E^2$.

Question: Is there a cograph-$(2,1)$ $G = (V, E)$ such that $E^1 \subseteq E \subseteq E^2$?

COGRAPH-$(2,1)$ - SP

COGRAPH-$(2,1)$ GRAPH SANDWICH PROBLEM-(COGRAPH-$(2,1)$-SP)

Input: $G^1 = (V, E^1)$ and $G^2 = (V, E^2)$ such that $E^1 \subseteq E^2$.

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COGRAPH-$(2,1)$ - SP

COGRAPH-$(2,1)$ GRAPH SANDWICH PROBLEM-(COGRAPH-$(2,1)$-SP)

Input: $G^1 = (V, E^1)$ and $G^2 = (V, E^2)$ such that $E^1 \subseteq E^2$.

Question: Is there a cograph-$(2,1)$ $G = (V, E)$ such that $E^1 \subseteq E \subseteq E^2$?

Final Result

Theorem 15

COGRAPH-$(2,1)$-SP *is NP-complete.*

Proof's Idea. Polynomial-time reduction from the NP-complete problem JTT-SP.

Final Result

Theorem 15

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Proof's Idea. Polynomial-time reduction from the NP-complete problem JTT-SP.

Full dichotomy

$k \setminus \ell$	0	1	2	3	4	...
0	-	P	P	NP-c	NP-c	...
1	P	P	NP-c	NP-c	NP-c	...
2	P	NP-c	NP-c	NP-c	NP-c	...
3	NP-c	NP-c	NP-c	NP-c	NP-c	...
4	NP-c	NP-c	NP-c	NP-c	NP-c	...
⋮	⋮	⋮	⋮	⋮	⋮	⋱

Table: Dichotomy P x NP-c of COGRAPH-(k, ℓ)-SP.

Characterization, probe and sandwich problems on a generalization of threshold graphs

Vinicius F. dos Santos
Universidade Federal de Minas Gerais (UFMG)

Joint work with Fernanda Couto, Luerbio Faria, Sylvain Gravier and Sulamita Klein

February, 2019

LAGOS 2019

Latin & American
Algorithms, Graphs and Optimization
Symposium

Belo Horizonte, Brazil, June 2nd - 7th

Confirmed Invited Speakers

- Karen Aardal (Delft University of Technology, Netherlands)
- Sebastian Cioabă (University of Delaware, USA)
- Michael Fellows (University of Bergen, Norway)
- Fabio Protti (UFF, Brazil)
- Ignasi Sau (CNRS, LIRMM, Université de Montpellier, France)
- Maya Stein (Universidad de Chile, Chile)
- Vilmar Trevisan (UFRGS, Brazil)
- Mario Valencia-Pabon (Université Paris-13, France)

Special session in honor of Frédéric Maffray

- Organized by Claudia Linhares (UFC, Brazil)