Geometry of Singularities

mini-workshop, Ben-Gurion University,

April 9, 2018.

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Participants Juan José Nuño Ballesteros (Universitat de València), J.Edson Sampaio (Universidade Federal do Ceará, Fortaleza), Marina Ville (Franҫois Rabelais, Tours), Michael Zhitomirsky (Technion), Sergei Malev (Technion), Yosef Yomdin (WIS), Dmitry Novikov (WIS), Genrich Belitskii (BGU), Alberto Fernandez Boix (BGU).

Organizers Lev Birbrair (Universidade Federal do Ceará, Fortaleza), Dmitry Kerner (BGU).

Location Math building, [58], room (-101).

Schedule

- 9:40-10:00 cofee-cookies
- 10:00-10:40, J.J.Nuño Ballesteros *Equisingularity of families of isolated determinantal singularities*
- 11:00-11:40, J.Edson Sampaio, *Tangent cones of Lipschitz normally embedded sets*
- 12:00-12:40, Marina Ville, *Branch points of minimal surfaces in* $\mathbb{R}^4$
- lunch break
- 14:30-15:10, D.Novikov, *Complex Cellular Structures*
- 15:30-16:10, Y.Yomdin, *Accuracy of noisy Spike-Train Reconstruction: a Singularity Theory point of view*

Abstracts

- J.J.Nuño Ballesteros *Equisingularity of families of isolated determinantal singularities*

  We study the topological triviality and the Whitney equisingularity of a family of isolated determinantal singularities. On one hand, we give a Lê-Ramanujam type theorem for this kind of singularities by using the homotopy type of the determinantal smoothing. On the other hand, we extend the results of Teissier and Gaffney about the Whitney equisingularity of hypersurfaces and complete intersections, respectively, in terms of the constancy of the polar multiplicities.
- J.Edson Sampaio, *Tangent cones of Lipschitz normally embedded sets*

  We show that tangent cones of Lipschitz normally embedded (LNE) sets are LNE and reduced. As an application, we can to prove that if a complex analytic set X is LNE and its tangent cone is a linear subspace, then X is smooth. Moreover, we show that if X is a complex algebraic set and Lipschitz regular at infinity, then X is affine linear subspace. This is a work jointly with Alexandre Fernandes.
- Marina Ville, *Branch points of minimal surfaces in* $\mathbb{R}^4$

  Minimal surfaces in $\mathbb{R}^4$ are a generalization of complex curves in $\mathbb{C}^2$. After recalling their definition, I will talk of their singularities, focusing on the branch points. When a branch point is isolated, we replicate Milnor's construction of algebraic knots and intersect the surface with a small 3-sphere around the branch point; we derive a class of knots called minimal knots generalizing algebraic knots. How large is this class is still a mystery but I will describe a sub-class of these knots called Lissajous toric knots: they consist in a Lissajous curve $(cos(pt), sin(qt+a))$ in a vertical plane in $\mathbb{R}^3$ rotating N times around the Oz axis. I will give a braid representation for these knots and explain why they are ribbon if N, p and q are all mutually prime. Finally I will describe two different ways of desingularizing a branch point of a minimal surface.
- D.Novikov *Complex Cellular Structures*, (joint with Gal Binyamini).

  We complexify the notion of real cell, and prove a complex analogue of a cellular decomposition. This decomposition has polynomial complexity in semialgebraic case, and is uniform in families in subanalytic case. The keystone of the proof is a new, Picard-Koebe style, geometric functions theory result. As a corollary, we provide an upper bound on tail entropy for analytic maps, settling an old conjecture of Yomdin from 1991, and prove some bounds on the number of rational points on algebraic and log-analytic sets.
- Y.Yomdin, *Accuracy of noisy Spike-Train Reconstruction: a Singularity Theory point of view*

  This is a survey talk discussing one specific (and classical) system of algebraic equations - the so called "Prony system". We provide a short overview of its unusually wide connections with many different fields of Mathematics, stressing the role of Singularity Theory. We reformulate Prony System as the problem of reconstruction of "Spike-train" signals of the form $F(x) = \sum_{j=1}^{d} a_j \delta(x - x_j)$ from the noisy moment measurements. We provide an overview of some recent results on the "geometry of the error amplification" in the reconstruction process, in situations where the nodes x_j near-collide. Some algebraic-geometric structures, underlying the error amplification, are described (Prony, Vieta, and Hankel mappings, Prony varieties), as well as their connection with Vandermonde mappings and varieties. Our main goal is to present some promising fields of possible applications of Singularity Theory.